LESSON 67

• Using Prime Factorization to Reduce Fractions

Planning & Preparation

OBJECTIVES

- **Use prime factorization to reduce fractions.**

MATERIALS

Instructional Masters

- Power Up J

MATH LANGUAGE

Maintain

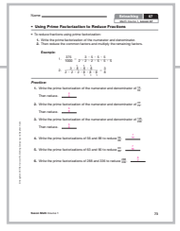
- reduce

English Learners (ESL)

- collage

Differentiated Instruction

Many resources are available to help meet the needs of diverse learners.



◀ *Lesson 67 Reteaching Master*

Inclusion	Intervention	English Learners	Advanced Learners
• Adaptations for Saxon Math	• Error Alert (TM, *pp. 347, 348*) • Reteaching Masters	• English Learners (TM, *p. 348*) • English Learner Handbook	• Extend the Problem (TM, *p. 348*)

TM = Teacher's Manual; SE = Student Edition

LESSON 67

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Power Up

1. Facts

Distribute **Power Up J** to students.

Name _____ Time _____

Power Up J
Use with Lesson 67

Facts Write each mixed number as an improper fraction.

$2\frac{1}{2} = \frac{5}{2}$	$2\frac{2}{5} = \frac{12}{5}$	$1\frac{3}{4} = \frac{7}{4}$	$2\frac{3}{4} = \frac{11}{4}$	$2\frac{1}{8} = \frac{17}{8}$
$1\frac{2}{3} = \frac{5}{3}$	$3\frac{1}{2} = \frac{7}{2}$	$1\frac{5}{6} = \frac{11}{6}$	$2\frac{1}{4} = \frac{9}{4}$	$1\frac{1}{8} = \frac{9}{8}$
$5\frac{1}{2} = \frac{11}{2}$	$1\frac{3}{8} = \frac{11}{8}$	$5\frac{1}{3} = \frac{16}{3}$	$3\frac{1}{4} = \frac{13}{4}$	$4\frac{1}{2} = \frac{9}{2}$
$1\frac{7}{8} = \frac{15}{8}$	$2\frac{2}{3} = \frac{8}{3}$	$1\frac{5}{8} = \frac{13}{8}$	$3\frac{3}{4} = \frac{15}{4}$	$7\frac{1}{2} = \frac{15}{2}$

Mental Math

a.	b.	c.	d.
e.	f.	g.	h.

Problem-Solving Discussion

FOCUS STRATEGY

Use Logical Reasoning

Copy this problem and fill in the missing digits.

$$\begin{array}{r} \_\_\_,\_\_\_ \\ \times \quad\quad 7 \\ \hline 999{,}999 \end{array}$$

UNDERSTAND ***Understand the problem.***

"What information are we given?"

We are shown a multiplication problem with six missing digits in the multiplicand.

"What are we asked to do?"

Fill in the missing digits.

PLAN ***Make a plan.***

"What problem-solving strategy will we use?"

We could *use logical reasoning* and number sense to find the missing digits.

"Is there anything we know about missing factors that can make the task easier?"

If we know the product and one factor, we can find the missing factor by dividing.

SOLVE ***Carry out the plan.***

"What numbers do we divide?"

We divide 999,999 by 7.

$$7\overline{)999{,}999}\quad 142,857$$

"How do we show the solution to the problem?"

$$\begin{array}{r} 142\ 857 \\ \_\_\_,\_\_\_ \\ \times \quad\quad 7 \\ \hline 999{,}999 \end{array}$$

CHECK ***Look back.***

"Did we complete the task?"

yes

"How can we check our answer?"

We can perform the multiplication to be sure the product is 999,999.

LESSON 67

Power Up *continued*

2. Mental Math

Before students begin the Mental Math exercise, do this counting exercise as a class.

Count down by 2s from 10 to negative 10.

Encourage students to share different ways to mentally compute these exercises. Strategies for exercises **b** and **d** are listed below.

b. Change 341 to 340, then Add 1

$340 - 50 = 290$; $290 + 1 = 291$

Change 341 to 350, then Subtract 9

$350 - 50 = 300$; $300 - 9 = 291$

d. Subtract $1, then Add 25¢

$\$9.25 - \$1 = \$8.25$; $\$8.25 + 25¢ = \$8.50$

Subtract 25¢ from Each Number

$\$9.00 - 50¢ = \$8.50$

3. Problem Solving

Refer to **Power-Up Discussion,** p. 346B.

New Concept

INSTRUCTION

A generalization students should make about factoring the terms of fractions is that a goal of the factoring is to identify common factors that are equivalent to 1.

Explain that the first step to complete when reducing fractions with large terms is to factor the numerator and factor the denominator. The factoring can be accomplished by making a factor tree or division by primes.

"Why is a 5 in the numerator and a 5 in the denominator changed to a 1 in the numerator and a 1 in the denominator?"

The factor pairs are equivalent to 1 over 1.

Example

INSTRUCTION

Ask students to describe how a calculator can be used to check the answer $\frac{3}{8}$. Divide 375 by 1000 and divide 3 by 8 to learn the decimal equivalent of each fraction, then compare. If the decimal equivalents are the same, the assumption can be made that the answer is correct.

LESSON 67

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Power Up

Facts Power Up J

Mental Math

a. Number Sense: 5×260 1300

b. Number Sense: $341 - 50$ 291

c. Calculation: 3×48 144

d. Calculation: $\$9.25 - 75¢$ $8.50

e. Number Sense: Double $1.25. $2.50

f. Number Sense: $\frac{\$30}{100}$ $0.30

g. Measurement: Which is greater, 3 yards or 5 feet? 3 yd

h. Calculation: $6 \times 6, -1, \div 5, \times 2, +1, \div 3, \div 2$ $2\frac{1}{2}$

Problem Solving Copy this problem and fill in the missing digits:

$$\begin{array}{r} ---,--- \\ \times \quad 7 \\ \hline 999,999 \end{array} \qquad \begin{array}{r} 142,857 \\ \times \quad 7 \\ \hline 999,999 \end{array}$$

New Concept

One way to **reduce** fractions with large terms is to factor the terms and then reduce the common factors. To reduce $\frac{125}{1000}$, we could begin by writing the prime factorizations of 125 and 1000.

$$\frac{125}{1000} = \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$$

We see three pairs of 5s that can be reduced. Each $\frac{5}{5}$ reduces to $\frac{1}{1}$.

$$\frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = \frac{1}{8}$$

We multiply the remaining factors and find that $\frac{125}{1000}$ reduces to $\frac{1}{8}$.

Thinking Skill

Explain

What are two strategies for finding the prime factorization of a number?

division by primes, factor trees

Thinking Skill

Discuss

When is it helpful to use prime factorization to reduce a fraction?

If the terms are large or if the GCF is not obvious.

Example

Reduce: $\frac{375}{1000}$

Solution

We write the prime factorization of both the numerator and the denominator.

$$\frac{375}{1000} = \frac{3 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{8}$$

Then we reduce the common factors and multiply the remaining factors.

Power Up J

Facts Write each mixed number as an improper fraction.

$2\frac{1}{2} = \frac{5}{2}$	$2\frac{2}{5} = \frac{12}{5}$	$1\frac{3}{4} = \frac{7}{4}$	$2\frac{3}{4} = \frac{11}{4}$	$2\frac{1}{8} = \frac{17}{8}$
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Lesson Practice Write the prime factorization of both the numerator and the denominator of each fraction. Then reduce each fraction.

a. $\frac{875}{1000}$ $\frac{5 \cdot 5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{7}{8}$

b. $\frac{48}{400}$ $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5} = \frac{3}{25}$

c. $\frac{125}{500}$ $\frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{4}$

d. $\frac{36}{81}$ $\frac{2 \cdot 2 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{4}{9}$

Written Practice

*1. (66) Allison is making a large collage of a beach scene. She needs 2 yards of blue ribbon for the ocean, $\frac{1}{2}$ yard of yellow ribbon for the sun, and $\frac{3}{4}$ yard of green ribbon for the grass. Ribbon costs $2 a yard. How much money will Allison need for ribbon? $6.50

2. (13) ESTIMATE A mile is 5280 feet. A nautical mile is about 6080 feet. A nautical mile is about how much longer than a mile? about 800 feet

3. (43) VERIFY Instead of dividing $1.50 by $0.05, Marcus formed an equivalent division problem by mentally multiplying both the dividend and the divisor by 100. Then he performed the equivalent division problem. What is the equivalent division problem Marcus formed, and what is the quotient? $150 ÷ $5 = 30 (not $30.00 or 30¢)

Find each unknown number:

4. (3) 6 cm + k = 11 cm 5 cm

5. (43) $8g = 9.6$ 1.2

6. (43) $\frac{7}{10} - w = \frac{1}{2}$ $\frac{1}{5}$

7. (42) $\frac{3}{5} = \frac{n}{100}$ 60

*8. (60, 64) The perimeter of a quadrilateral is 172 inches. What is the average length of each side? Can we know for certain what type of quadrilateral this is? Why or why not?

8. 43 inches; No; There is not enough given information to know the type of quadrilateral. Many combinations of four side lengths total 172 inches and can form a quadrilateral.

9. (5) $100.00 − ($46.75 + $9.68) $43.57

10. (53) (2 × 0.3) − (0.2 × 0.3) 0.54

*11. (63) ANALYZE $4\frac{1}{4} - 2\frac{7}{8}$ $1\frac{3}{8}$

*12. (38, 66) ANALYZE $2\frac{2}{3} \times \sqrt{9}$ 8

13. (59) $3\frac{1}{3} + 2\frac{3}{4}$ $6\frac{1}{12}$

*14. (66) $1\frac{1}{3} \times 2\frac{1}{4}$ 3

15. (45) 1.44 ÷ 60 0.024

16. (49) $6.00 ÷ $0.15 40

17. (15) Five dollars was divided evenly among 4 people. How much money did each receive? $1.25

MATH BACKGROUND

Factoring the terms of a fraction, then identifying common factors that are equivalent to 1, produces the same reduced fraction as dividing the numerator and the denominator of a fraction by their greatest common factor (GCF).

For example, students are shown in this lesson how factoring is used to reduce $\frac{125}{1000}$ to its simplest form of $\frac{1}{8}$. The same result can be achieved by dividing $\frac{125}{1000}$ by $\frac{125}{125}$, because 125 is the GCF of 125 and 1000.

In their future studies of polynomials and other algebraic expressions, students will frequently factor out the GCF of a variety of terms.

New Concept *continued*

Lesson Practice

Problems a–d ERROR ALERT

To give students practice finding prime factorizations using different methods, have them divide by primes for problems a and c and make factor trees for problems b and d.

Written Practice

Math Conversations

Discussion opportunities are provided below.

Problem 11 ANALYZE

"Before you can subtract mixed numbers whose fraction parts have unlike denominators, what must you do?" Choose a common denominator and rename one or both fractions using that denominator.

"What is the least common denominator of 4 and 8?" 8

Problem 12 ANALYZE

"How is the Order of Operations used to simplify this expression?" The Order of Operations states that the square root of 9 must be simplified before the factors are multiplied.

Math Conversations

Discussion opportunities are provided below.

Problem 23 ANALYZE

"What is the product of any number and its reciprocal?" 1

"If the missing number is the reciprocal of $1\frac{1}{2}$, will the product of the factors be 1?" yes

"What should we do to the mixed number $1\frac{1}{2}$ to help find its reciprocal?" change it to an improper fraction

Problem 26 ESTIMATE

EXTEND THE PROBLEM

"Explain how to estimate the number of hours it would take the minute hand to travel one mile." Sample: 38 inches is about 1 yard, and there are 1760 yards in 1 mile, so it takes the minute hand about 1760 hours to travel a mile.

Point out that 1760 hours is about 73 days.

Problem 29 REPRESENT

"How many faces does a number cube have?" six

"How many dots are on the faces of the cube that you cannot see?" two, four, and six

Errors and Misconceptions

Problem 18 CONCLUDE

If the word "regular" is overlooked, there is not enough information remaining in the problem to solve it successfully.

The following questions can be used to help students identify the figure as a square.

"Some polygons are regular, and some are not. What is a regular polygon?" All of the sides of the polygon have the same length and all of the angles of the polygon have the same measure.

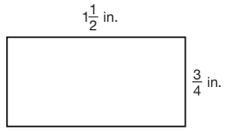
"A quadrilateral has 4 sides. What kind of quadrilateral is a regular quadrilateral?" a square

LESSON 67

18. (60) CONCLUDE The area of a regular quadrilateral is 100 square inches. What is its perimeter? What is the name of the quadrilateral? 40 inches; square

***19.** (67) Write the prime factorizations of 625 and of 1000. Then reduce $\frac{625}{1000}$. $\frac{5 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{5}{8}$

***20.** (31, 66) What is the area of the rectangle shown below? $1\frac{1}{8}$ in.^{2}



21. (29) Thirty-six of the 88 piano keys are black. What fraction of the piano keys are black? $\frac{9}{22}$

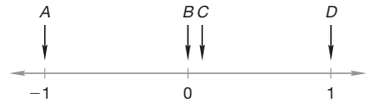
22. Sample:

***22.** (Inv. 6) REPRESENT Draw a rectangular prism. Begin by drawing two congruent rectangles.

***23.** (30, 62) ANALYZE $1\frac{1}{2} \times \square = 1$ $\frac{2}{3}$

24. (15) There are 1000 meters in a kilometer. How many meters are in 2.5 kilometers? 2500 meters

25. (50) CONNECT Which arrow could be pointing to 0.1 on the number line? C



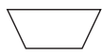
26. (47) ESTIMATE If the tip of the minute hand is 6 inches from the center of the clock, how far does the tip travel in one hour? Round the answer to the nearest inch. (Use 3.14 for π.) 38 inches

27. (Inv. 6) CONNECT A basketball is an example of what geometric solid? sphere

28. (33, 35) Write 51% as a fraction. Then write the fraction as a decimal number. $\frac{51}{100}$; 0.51

***29.** (19, 58) REPRESENT What is the probability of rolling a prime number with one toss of a number cube? The probability of rolling a 2, 3, or 5 is $\frac{1}{2}$.

***30.** (64) CONCLUDE This quadrilateral has one pair of parallel sides. What kind of quadrilateral is it? trapezoid



ENGLISH LEARNERS

Refer students to problem 1. Write the word ***collage*** on the board. Say, ***"A collage is a collection of things such as paper, ribbon, and pictures glued to a flat surface."***

Ask a student to list items they might include in a collage of a beach scene.

LOOKING FORWARD

Using prime factorization to reduce fractions prepares students for:

- **Lesson 68,** dividing mixed numbers.
- **Lesson 70,** dividing fractions by reducing fraction terms before multiplying.
- **Lesson 72,** using a fractions chart to recall operations with fractions and reducing fractions.
- **Lesson 75,** reducing fractions when writing fractions as percents.